

# **XXV EVENTO NACIONAL ESTUDIANTIL DE CIENCIAS. ENEC 2018.**

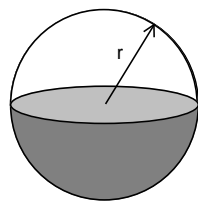
## **FORMULARIO DE CIENCIAS BÁSICAS**

### **MATEMÁTICAS**

## GEOMETRÍA

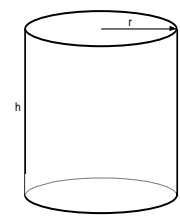
$$\text{Volumen} = \frac{4}{3} \pi r^3$$

$$\text{Área de la Superficie} = 4 \pi r^2$$



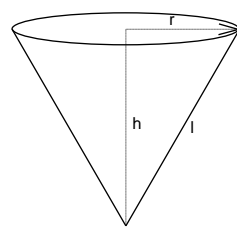
$$\text{Volumen} = \pi r^2 h$$

$$\text{Área de la superficie lateral} = 2 \pi r h$$

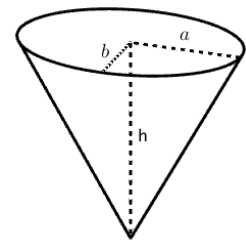


$$\text{Volumen} = \frac{1}{3} \pi r^2 h$$

$$\text{Área de la superficie lateral} = \pi r \sqrt{r^2 + h^2} = \pi r l$$

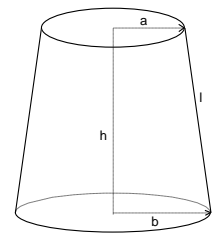


$$\text{Volumen} = \frac{1}{3} \pi a b h$$



$$\text{Volumen} = \frac{1}{3} \pi h (a^2 + a b + b^2)$$

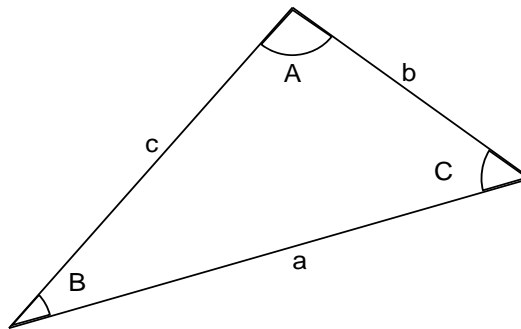
$$\begin{aligned} \text{Área de la superficie lateral} &= \pi (a + b) \sqrt{h^2 + (b - a)^2} \\ &= \pi (a + b) l \end{aligned}$$



## TRIGONOMETRÍA

$\text{sen}^2 A + \text{cos}^2 A = 1$	$\text{sen}^2 A = \frac{1}{2} - \frac{1}{2} \text{cos} 2A$
$\text{sec}^2 A - \text{tan}^2 A = 1$	$\text{cos}^2 A = \frac{1}{2} + \frac{1}{2} \text{cos} 2A$
$\text{csc}^2 A - \text{cot}^2 A = 1$	$\text{sen} 2A = 2 \text{sen} A \text{cos} A$
$\text{sen} A \text{csc} A = 1$	$\text{cos} 2A = \text{cos}^2 A - \text{sen}^2 A$
$\text{cos} A \text{sec} A = 1$	$\text{sen} (A \pm B) = \text{sen} A \text{cos} B \pm \text{cos} A \text{sen} B$
$\text{tan} A \text{cot} A = 1$	$\text{cos} (A \pm B) = \text{cos} A \text{cos} B \mp \text{sen} A \text{sen} B$
$\text{sen} (-A) = -\text{sen} A$	$\text{tan} (A \pm B) = \frac{\text{tan} A \pm \text{tan} B}{1 \mp \text{tan} A \text{tan} B}$
$\text{cos} (-A) = \text{cos} A$	$\text{sen} \frac{A}{2} = \pm \sqrt{\frac{1 - \text{cos} A}{2}}$
$\text{tan} (-A) = -\text{tan} A$	$\text{cos} \frac{A}{2} = \pm \sqrt{\frac{1 + \text{cos} A}{2}}$
$\text{sen} A \text{cos} B = \frac{1}{2} [\text{sen} (A - B) + \text{sen} (A + B)]$	$\text{sen} A \text{sen} B = \frac{1}{2} [\text{cos} (A - B) - \text{cos} (A + B)]$
	$\text{cos} A \text{cos} B = \frac{1}{2} [\text{cos} (A - B) + \text{cos} (A + B)]$

Sea el siguiente triángulo plano  $ABC$  de lados  $a, b, c$  y ángulos  $A, B, C$ .



Ley de los senos	$\frac{a}{\text{sen} A} = \frac{b}{\text{sen} B} = \frac{c}{\text{sen} C}$	
Ley de los cosenos	$c^2 = a^2 + b^2 - 2ab \text{cos} C$	Los otros lados y ángulos están relacionados en forma similar
Ley de las tangentes	$\frac{a+b}{a-b} = \frac{\text{tan} \frac{1}{2}(A+B)}{\text{tan} \frac{1}{2}(A-B)}$	Los otros lados y ángulos están relacionados en forma similar

## NÚMEROS COMPLEJOS

Teorema de DeMoivre	$[r(\text{cos} \theta + i \text{sen} \theta)]^n = r^n (\text{cos} n\theta + i \text{sen} n\theta)$	$n$ : número entero
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Raíz compleja	$[r(\cos\theta + isen\theta)]^{\frac{1}{n}} = r^{\frac{1}{n}} \left[ \cos\left(\frac{\theta+2k\pi}{n}\right) + isen\left(\frac{\theta+2k\pi}{n}\right) \right]$	número entero positivo $k = 0, 1, 2, \dots, n-1$
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## GEOMETRÍA ANALÍTICA DEL ESPACIO

Considerando  $P_1(x_1, y_1, z_1)$  y  $P_2(x_2, y_2, z_2)$ :

Vector que une $P_1$ y $P_2$	$\overline{P_1P_2} = \langle (x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \rangle = \langle l, m, n \rangle$
Distancia entre dos puntos	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{l^2 + m^2 + n^2}$
Recta que pasa por dos puntos	Forma paramétrica $x = x_1 + lt \quad y = y_1 + mt \quad z = z_1 + nt$
	Forma simétrica $t = \frac{x - x_1}{l} \quad t = \frac{y - y_1}{m} \quad t = \frac{z - z_1}{n}$
Cosenos Directores	$\cos\alpha = \frac{x_2 - x_1}{d} = \frac{l}{d} \quad \cos\beta = \frac{y_2 - y_1}{d} = \frac{m}{d} \quad \cos\gamma = \frac{z_2 - z_1}{d} = \frac{n}{d}$
	donde $\alpha, \beta, \gamma$ ángulos que forman la línea que une los puntos $P_1$ y $P_2$ con la parte positiva de los ejes $x, y, z$ , respectivamente
	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad l^2 + m^2 + n^2 = 1$
Ecuación del Plano	Que pasa por un punto $P_1(x_1, y_1, z_1)$ y tiene vector normal $\vec{n} = \langle n_1, n_2, n_3 \rangle$ $n_1(x - x_1) + n_2(y - y_1) + n_3(z - z_1) = 0$
	Forma general $Ax + By + Cz + D = 0$
	Distancia del punto $P_0(x_0, y_0, z_0)$ al plano $Ax + By + Cz + D = 0$ $d = \frac{ Ax_0 + By_0 + Cz_0 + D }{\sqrt{A^2 + B^2 + C^2}}$
	Ángulo entre dos rectas en el plano $\tan\alpha = \frac{m_2 - m_1}{1 + m_1m_2}$

Coordenadas:

<p>Cilíndricas <math>(r, \theta, z)</math></p> $\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases} \quad \text{o} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \\ z = z \end{cases}$	
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<p>Esféricas <math>(r, \theta, \phi)</math></p> $\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases} \quad \text{o} \quad \begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \tan^{-1}\left(\frac{y}{x}\right) \text{ con } x \neq 0 \\ \theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) \end{cases}$	
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## REGLAS GENERALES DE DERIVACIÓN

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(uvw) = uv \frac{dw}{dx} + uw \frac{dv}{dx} + vw \frac{du}{dx}$
$\frac{d}{dx}(cx) = c$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$
$\frac{d}{dx}(cx^n) = ncx^{n-1}$	$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$
$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$	$\frac{dF}{dx} = \frac{dF}{du} \frac{du}{dx}$ (Regla de la cadena)
$\frac{d}{dx}(cu) = c \frac{du}{dx}$	$\frac{du}{dx} = \frac{1}{\frac{dx}{du}}$

$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	$\frac{dF}{dx} = \frac{\frac{dF}{du}}{\frac{dx}{du}}$
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### Derivadas de las Funciones Exponenciales y Logarítmicas

$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx} \quad a > 0, \quad a \neq 1$
$\frac{d}{dx} \ln u = \frac{d}{dx} \log_e u = \frac{1}{u} \frac{du}{dx}$
$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$
$\frac{d}{dx} e^u = e^u \frac{du}{dx}$
$\frac{d}{dx} u^v = \frac{d}{dx} e^{v \ln u} = e^{v \ln u} \frac{d}{dx} [v \ln u] = v u^{v-1} \frac{du}{dx} + u^v \ln u \frac{dv}{dx}$

### Derivadas de las Funciones Trigonométricas y de las Trigonométricas Inversas

$\frac{d}{dx} \operatorname{sen} u = \operatorname{cos} u \frac{du}{dx}$	$\frac{d}{dx} \operatorname{cos} u = -\operatorname{sen} u \frac{du}{dx}$
$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$	$\frac{d}{dx} \cot u = -\operatorname{csc}^2 u \frac{du}{dx}$
$\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$	$\frac{d}{dx} \operatorname{csc} u = -\operatorname{csc} u \cot u \frac{du}{dx}$
$\frac{d}{dx} \operatorname{sen}^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \quad \left[ -\frac{\pi}{2} < \operatorname{sen}^{-1} u < \frac{\pi}{2} \right]$	$\frac{d}{dx} \operatorname{cos}^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx} \quad [0 < \operatorname{cos}^{-1} u < \pi]$
$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} \quad \left[ -\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2} \right]$	$\frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \frac{du}{dx} \quad [0 < \cot^{-1} u < \pi]$
$\frac{d}{dx} \sec^{-1} u = \frac{1}{ u  \sqrt{u^2-1}} \frac{du}{dx} = \frac{\pm 1}{u \sqrt{u^2-1}} \frac{du}{dx}$	
$\left[ \begin{array}{l} + \text{ si } 0 < \sec^{-1} u < \frac{\pi}{2} \\ - \text{ si } \frac{\pi}{2} < \sec^{-1} u < \pi \end{array} \right]$	

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$$\left[ \begin{array}{l} - \text{ si } 0 < \csc^{-1} u < \frac{\pi}{2} \\ + \text{ si } -\frac{\pi}{2} < \csc^{-1} u < 0 \end{array} \right]$$

$$\frac{d}{dx} \csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}} \frac{du}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}} \frac{du}{dx}$$

**Derivadas de las Funciones Hiperbólicas y de las Hiperbólicas Recíprocas**

$\frac{d}{dx} \sinh u = \cosh u \frac{du}{dx}$	$\frac{d}{dx} \cosh u = \sinh u \frac{du}{dx}$
$\frac{d}{dx} \tanh u = \text{sech}^2 u \frac{du}{dx}$	$\frac{d}{dx} \coth u = -\text{csch}^2 u \frac{du}{dx}$
$\frac{d}{dx} \text{sech} u = -\text{sech} u \tanh u \frac{du}{dx}$	$\frac{d}{dx} \text{csch} u = -\text{csch} u \coth u \frac{du}{dx}$
$\frac{d}{dx} \text{senh}^{-1} u = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$	$\frac{d}{dx} \text{cosh}^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}} \frac{du}{dx} \left[ \begin{array}{l} + \text{ si } \text{cosh}^{-1} u > 0, u > 1 \\ - \text{ si } \text{cosh}^{-1} u < 0, u < 1 \end{array} \right]$
$\frac{d}{dx} \tanh^{-1} u = \frac{1}{1-u^2} \frac{du}{dx} \quad [-1 < u < 1]$	$\frac{d}{dx} \coth^{-1} u = \frac{1}{1-u^2} \frac{du}{dx} \quad [u > 1 \text{ o } u < -1]$
$\frac{d}{dx} \text{sech}^{-1} u = \frac{\pm 1}{u\sqrt{u^2-1}} \frac{du}{dx}$	$\left[ \begin{array}{l} - \text{ si } \text{sech}^{-1} u > 0, 0 < u < 1 \\ + \text{ si } \text{sech}^{-1} u < 0, 0 < u < 1 \end{array} \right]$
$\frac{d}{dx} \text{csch}^{-1} u = \frac{-1}{ u \sqrt{1+u^2}} \frac{du}{dx} = \frac{\mp 1}{u\sqrt{1+u^2}} \frac{du}{dx}$	$\left[ \begin{array}{l} - \text{ si } u > 0 \\ + \text{ si } u < 0 \end{array} \right]$

## TABLAS DE INTEGRALES

$$\int u dv = uv - \int v du$$

$$\int u^n du = \frac{1}{n+1} u^{n+1} + C \quad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \operatorname{sen} u du = -\operatorname{cos} u + C$$

$$\int \operatorname{cos} u du = \operatorname{sen} u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$\int \tan u du = \ln|\sec u| + C$$

$$\int \cot u du = \ln|\operatorname{sen} u| + C$$

$$\int \sec u du = \ln|\sec u + \tan u| + C$$

$$\int \csc u du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln|u + \sqrt{a^2 + u^2}| + C$$

$$\int u^2 \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^2}{8} \ln|u + \sqrt{a^2 + u^2}| + C$$

$$\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$$

$$\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$$



$$\int \frac{\sqrt{a^2+u^2}}{u} du = \sqrt{a^2+u^2} - a \ln \left| \frac{a+\sqrt{a^2+u^2}}{u} \right| + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u^2} du = -\frac{\sqrt{a^2+u^2}}{u} + \ln |u+\sqrt{a^2+u^2}| + C$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln |u+\sqrt{a^2+u^2}| + C$$

$$\int \frac{u^2 du}{\sqrt{a^2+u^2}} = \frac{u}{2} \sqrt{a^2+u^2} - \frac{a^2}{2} \ln |u+\sqrt{a^2+u^2}| + C$$

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{1}{a^2 \sqrt{a^2+u^2}} + C$$

$$\int \sqrt{a^2-u^2} du = \frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \text{sen}^{-1} \frac{u}{a} + C$$

$$\int u^2 \sqrt{a^2-u^2} du = \frac{u}{8} (2u^2-a^2) \sqrt{a^2-u^2} + \frac{a^4}{8} \text{sen}^{-1} \frac{u}{a} + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u} du = \sqrt{a^2-u^2} - a \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2-u^2} - \text{sen}^{-1} \frac{u}{a} + C$$

$$\int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \text{sen}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u \sqrt{a^2-u^2}} = -\frac{1}{a} \ln \left| \frac{a+\sqrt{a^2-u^2}}{u} \right| + C$$

$$\int \frac{du}{u^2 \sqrt{a^2-u^2}} = -\frac{1}{a^2 u} \sqrt{a^2-u^2} + C$$

$$\int (a^2-u^2)^{3/2} du = -\frac{u}{8} (2u^2-5a^2) \sqrt{a^2-u^2} + \frac{3a^4}{8} \text{sen}^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2-u^2}} + C$$

$$\int \frac{udu}{a+bu} = \frac{1}{b^2} (a+bu - a \ln |a+bu|) + C$$

$$\int \frac{u^2 du}{a+bu} = \frac{1}{2b^3} [(a+bu)^2 - 4a(a+bu) + 2a^2 \ln |a+bu|] + C$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a} \ln \left| \frac{u}{a+bu} \right| + C$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

$$\int \sqrt{u^2-a^2} du = \frac{u}{2} \sqrt{u^2-a^2} - \frac{a^2}{2} \ln |u+\sqrt{u^2-a^2}| + C$$

$$\int u^2 \sqrt{u^2-a^2} du = \frac{u}{8} (2u^2-a^2) \sqrt{u^2-a^2} - \frac{a^4}{8} \ln |u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u} du = \sqrt{u^2-a^2} - a \cos^{-1} \frac{a}{u} + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2} du = -\frac{\sqrt{u^2-a^2}}{u} + \ln |u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln |u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{u^2 du}{\sqrt{u^2-a^2}} = \frac{u}{2} \sqrt{u^2-a^2} + \frac{a^2}{2} \ln |u+\sqrt{u^2-a^2}| + C$$

$$\int \frac{du}{u^2 \sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2 u} + C$$

$$\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2-a^2}} + C$$

$$\int \frac{u^2 du}{\sqrt{a+bu}} = \frac{2}{15b^3} (8a^2+3b^2u^2-4abu) \sqrt{a+bu}$$

$$\int \frac{du}{u \sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}} \right| + C, \text{ si } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bu}{-a}} + C, \text{ si } a < 0$$

$$\int \frac{udu}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b} \ln|a+bu| + C$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2} \ln \left| \frac{a+bu}{u} \right| + C$$

$$\int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3} \left( a+bu - \frac{a^2}{a+bu} - 2a \ln|a+bu| \right) + C$$

$$\int u\sqrt{a+bu} du = \frac{2}{15b^2} (3bu-2a)(a+bu)^{\frac{3}{2}} + C$$

$$\int \frac{udu}{\sqrt{a+bu}} = \frac{2}{3b^2} (bu-2a)\sqrt{a+bu}$$

$$\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4} \sin 2u + C$$

$$\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4} \sin 2u + C$$

$$\int \tan^2 u du = \tan u - u + C$$

$$\int \cot^2 u du = -\cot u - u + C$$

$$\int \sin^3 u du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$$

$$\int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

$$\int \tan^3 u du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$$

$$\int \cot^3 u du = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$$

$$\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$$

$$\int \sin au \cos bu du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$\int u \sin u du = \sin u - u \cos u + C$$

$$\int u \cos u du = \cos u + u \sin u + C$$

$$\int u^n \sin u du = u^n \cos u + n \int u^{n-1} \cos u du$$

$$\int u^n \sqrt{a+bu} du = \frac{2}{b(2n+3)} \left[ u^n (a+bu)^{\frac{3}{2}} - na \int u^{n-1} \sqrt{a+bu} du \right]$$

$$\int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n \sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{u^{n-1} du}{\sqrt{a+bu}}$$

$$\int \frac{du}{u^n \sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a+bu}}$$

$$\int \csc^3 u du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$$

$$\int \sin^n u du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u du$$

$$\int \cos^n u du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u du$$

$$\int \tan^n u du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u du$$

$$\int \cot^n u du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u du$$

$$\int \sec^n u du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u du$$

$$\int \csc^n u du = \frac{1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u du$$

$$\int \sin au \sin bu du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

$$\int \cos au \cos bu du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

$$\int u^n \cos u du = u^n \sin u - n \int u^{n-1} \sin u du$$

$$\int \sin^n u \cos^m u du$$

$$= -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u du$$

$$= -\frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u du$$

$$\int u \cos^{-1} u du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u \tan^{-1} u du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$\int \operatorname{sen}^{-1} u \, du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$\int \operatorname{cos}^{-1} u \, du = u \operatorname{cos}^{-1} u - \sqrt{1-u^2} + C$$

$$\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$$

$$\int u \operatorname{sen}^{-1} u \, du = \frac{2u^2-1}{4} \operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int u e^{au} \, du = \frac{1}{a^2} (au-1) e^{au} + C$$

$$\int u^n e^{au} \, du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du$$

$$\int e^{au} \operatorname{sen} bu \, du = \frac{e^{au}}{a^2+b^2} (a \operatorname{sen} bu - b \operatorname{cos} bu) + C$$

$$\int e^{au} \operatorname{cos} bu \, du = \frac{e^{au}}{a^2+b^2} (a \operatorname{cos} bu + b \operatorname{sen} bu) + C$$

$$\int u^n \operatorname{sen}^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \operatorname{sen}^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$\int u^n \operatorname{cos}^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \operatorname{cos}^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1+u^2}} \right], n \neq -1$$

$$\int \ln u \, du = u \ln u - u + C$$

$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$\int \frac{1}{u \ln u} \, du = \ln |\ln u| + C$$

$$\int \operatorname{senh} u \, du = \operatorname{cosh} u + C$$

$$\int \operatorname{cosh} u \, du = \operatorname{senh} u + C$$

$$\int \tanh u \, du = \ln \operatorname{cosh} u + C$$

$$\int \operatorname{coth} u \, du = \ln |\operatorname{senh} u| + C$$

$$\int \operatorname{sech} u \, du = \tan^{-1} |\operatorname{senh} u| + C$$

$$\int \operatorname{sech} u \, du = \ln \left| \tan \frac{1}{2} u \right| + C$$

$$\int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$\int \operatorname{csch}^2 u \, du = -\operatorname{coth} u + C$$

$$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \operatorname{coth} u \, du = -\operatorname{csch} u + C$$

$$\int \sqrt{2au-u^2} \, du = \frac{u-a}{2} \sqrt{2au-u^2} + \frac{a^2}{2} \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int u \sqrt{2au-u^2} \, du = \frac{2u-au-3a^2}{6} \sqrt{2au-u^2} + \frac{a^3}{2} \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2} \, du = \sqrt{2au-u^2} + a \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{\sqrt{2au-u^2}}{u^2} \, du = -\frac{2\sqrt{2au-u^2}}{u} - \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{u^2 \, du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2} \sqrt{2au-u^2} + \frac{3a^2}{2} \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{du}{\sqrt{2au-u^2}} = \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{u \, du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a \operatorname{cos}^{-1} \left( \frac{a-u}{a} \right) + C$$

$$\int \frac{du}{u \sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$



## VECTORES

Producto punto	$ \mathbf{A} \cdot \mathbf{B}  = \ \mathbf{A}\  \ \mathbf{B}\  \cos \theta \quad 0 \leq \theta \leq \pi$ <p>donde <math>\theta</math> es el ángulo formado por <math>\mathbf{A}</math> y <math>\mathbf{B}</math></p>
	$\mathbf{A} \cdot \mathbf{B} = A_1 B_1 + A_2 B_2 + A_3 B_3$ <p>donde <math>\mathbf{A} = \langle A_1, A_2, \dots, A_n \rangle</math> y <math>\mathbf{B} = \langle B_1, B_2, \dots, B_n \rangle</math></p>
Producto cruz	$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$
	$= (A_2 B_3 - A_3 B_2) \hat{i} + (A_3 B_1 - A_1 B_3) \hat{j} + (A_1 B_2 - A_2 B_1) \hat{k}$ <p>donde <math>\mathbf{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}</math> y <math>\mathbf{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}</math></p>
	<p>Magnitud del producto cruz</p> $\ \mathbf{A} \times \mathbf{B}\  = \ \mathbf{A}\  \ \mathbf{B}\  \sin \theta$

Sean  $U = U(x, y, z)$ , una función escalar, y  $\mathbf{A} = \mathbf{A}(x, y, z)$ , una función vectorial, ambas con derivadas parciales

Operador nabla	$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$
Gradiente de $U$	$\text{grad } U = \nabla U = \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) U = \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k}$
Laplaciano de $U$	$\nabla^2 U = \nabla \cdot (\nabla U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$
Divergencia de $\mathbf{A}$	$\text{div } \mathbf{A} = \nabla \cdot \mathbf{A} = \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k})$ $= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$
Rotacional de $\mathbf{A}$	$\text{rot } \mathbf{A} = \nabla \times \mathbf{A} = \left( \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \times (A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k})$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$ $= \left( \frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z} \right) \hat{i} + \left( \frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x} \right) \hat{j} + \left( \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \right) \hat{k}$



## INTEGRALES MÚLTIPLES

Integrales dobles o integrales de área 	$\int_{x=a}^b \int_{y=f_1(x)}^{f_2(x)} F(x, y) dy dx = \int_{x=a}^b \left\{ \int_{y=f_1(x)}^{f_2(x)} F(x, y) dy \right\} dx$
	$\int_{y=c}^d \int_{x=g_1(y)}^{g_2(y)} F(x, y) dx dy = \int_{y=c}^d \left\{ \int_{x=g_1(y)}^{g_2(y)} F(x, y) dx \right\} dy$

Los anteriores conceptos se pueden ampliar para considerar integrales triples o de volumen así como integrales múltiples en más de tres dimensiones.

	En parámetro arbitrario:	En parámetro s:
Vector tangente unitario	$\hat{t}(t) = \frac{\vec{r}'(t)}{\ \vec{r}'(t)\ }$	$\hat{t}(s) = \frac{\dot{\vec{r}}(s)}{\ \dot{\vec{r}}(s)\ }$
Vector normal principal	$\hat{n}(t) = \hat{b}(t) \times \hat{t}(t)$	$\hat{n}(s) = \frac{\ddot{\vec{r}}(s)}{\ \ddot{\vec{r}}(s)\ }$
Vector binormal	$\hat{b}(t) = \frac{\vec{r}' \times \vec{r}''(t)}{\ \vec{r}' \times \vec{r}''(t)\ }$	$\hat{b}(s) = \frac{\dot{\vec{r}}(s) \times \ddot{\vec{r}}(s)}{\ \dot{\vec{r}}(s) \times \ddot{\vec{r}}(s)\ }$
Los vectores unitarios $\hat{t}, \hat{n}, \hat{b}$ guardan la relación $\hat{b} = \hat{t} \times \hat{n}$ , $\hat{n} = \hat{b} \times \hat{t}$ , $\hat{t} = \hat{n} \times \hat{b}$		

Recta tangente en $t_0$	Ecuación vectorial $\vec{r}(\lambda) = \vec{r}(t_0) + \lambda \vec{r}'(t_0)$
	Ecuación paramétrica $\frac{x - x_0}{x'_0} = \frac{y - y_0}{y'_0} = \frac{z - z_0}{z'_0}$
Plano osculador $(\hat{t}, \hat{n})$ en $t_0$	Ecuación vectorial $[\vec{r} - \vec{r}(t_0)] \cdot [\vec{r}'(t_0) \times \vec{r}''(t_0)] = 0$
	Ecuación paramétrica $\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x'_0 & y'_0 & z'_0 \\ x''_0 & y''_0 & z''_0 \end{vmatrix} = 0$
Plano normal	Ecuación vectorial $(\vec{r} - \vec{r}(t_0)) \cdot \vec{r}'(t_0) = 0$
	Ecuación paramétrica $x'_0(x - x_0) + y'_0(y - y_0) + z'_0(z - z_0) = 0$
Plano Rectificante $(\hat{t}, \hat{b})$ en $t_0$	Ecuación vectorial $(\vec{r} - \vec{r}(t_0)) \cdot \hat{n}(t_0) = 0$

	$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ x'_0 & y'_0 & z'_0 \\ y'_0 z''_0 - y''_0 z'_0 & z'_0 x''_0 - z''_0 x'_0 & x'_0 y''_0 - x''_0 y'_0 \end{vmatrix} = 0$
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Curvatura y Torsión	$\kappa(t) = \frac{\ \vec{r}'(t) \times \vec{r}''(t)\ }{\ \vec{r}'(t)\ ^3}$ $\tau(t) = \frac{\vec{r}'(t) \cdot [\vec{r}''(t) \times \vec{r}'''(t)]}{\ \vec{r}'(t) \times \vec{r}''(t)\ ^2}$ $\kappa(s) = \ \ddot{\vec{r}}(s)\  \qquad \kappa = \frac{ f''(x) }{[1 + (f'(x))^2]^{3/2}}$
Componentes Tangencial de la Aceleración	$a_T = \vec{a} \cdot \vec{T} = \frac{\vec{v} \cdot \vec{a}}{\ \vec{v}\ }$
Componentes Normal de la Aceleración	$a_N = \vec{a} \cdot \vec{N} = \frac{\ \vec{v} \times \vec{a}\ }{\ \vec{v}\ }$
Propiedades de la Divergencia	$\nabla \cdot (\vec{F} + \vec{G}) = \nabla \cdot \vec{F} + \nabla \cdot \vec{G}$ $\nabla \cdot (\phi \vec{F}) = \phi \nabla \cdot \vec{F} + (\nabla \phi) \cdot \vec{F}$ $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G})$



## TRANSFORMADA DE LAPLACE

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

No	$f(t)$	$F(s)$
1	$C$ (constante)	$\frac{C}{s}$
2	$t^n$	$\frac{n!}{s^{n+1}}$ , $n = 0$ y $n \in \mathbb{N}$
3	$t^n$	$\frac{\Gamma(n+1)}{s^{n+1}}$ , $n > -1$
4	$e^{at}$	$\frac{1}{s-a}$
5	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
6	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
7	$\sin(kt)$	$\frac{k}{s^2 + k^2}$
8	$\cos(kt)$	$\frac{s}{s^2 + k^2}$
9	$e^{at} f(t)$	$F(s-a)$
10	$f(t-a)U(t-a)$	$e^{-as} F(s)$
11	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
12	$\frac{f(t)}{t}$	$\int_s^{\infty} F(p) dp$
13	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$
14	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
15	$f * g = \int_0^t f(\tau) g(t-\tau) d\tau$	$F(s)G(s)$
16	$f(t)$ función periódica de periodo $T$	$\frac{1}{1 - e^{-sT}} \int_0^T f(t) e^{-st} dt$

17	$\delta(t)$	1	Educación e Innovación Innovación Educativa
18	$\delta(t - t_0)$	$e^{-t_0 s}$	

## FÓRMULAS MISCELÁNEAS

Área en coordenadas polares	$\frac{1}{2} \int_{\alpha}^{\beta} r^2 dr$
Ecuaciones paramétricas de la cicloide para $t \in R$	$x = a(t - \text{sen } t)$ $y = a(1 - \text{cost})$
Trabajo	$W = \int_a^b \vec{F} \cdot d\vec{r}$ $\text{Comp}(\vec{a}_b) = \frac{ \vec{a} \cdot \vec{b} }{\ \vec{b}\ }$
Longitud de arco de $y = f(x)$ en $[a, b] = \int_a^b \sqrt{1 + (y')^2} dx$	$m = \iint_R \rho(x, y) dA$ $M_x = \iint_R y \rho(x, y) dA$ $M_y = \iint_R x \rho(x, y) dA$
Centro de gravedad de una región plana	$\bar{x} = \frac{\int_a^b x f(x) dx}{\int_a^b f(x) dx}$ $\bar{y} = \frac{\frac{1}{2} \int_a^b [f(x)]^2 dx}{\int_a^b f(x) dx}$
Longitud de arco en forma paramétrica	$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
Momento de inercia de $R$ respecto al origen	$I_o = \iint_R (x^2 + y^2) \rho(x, y) dA$
Área de la superficie generada al girar la gráfica $f$ alrededor de $x$	$S = \int_a^b 2\pi F(x) \sqrt{1 + (f'(x))^2} dx$
Volumen del sólido de revolución generado al girar la gráfica de $f$ alrededor del eje $y$	$V = \int_a^b 2\pi t F(t) dt$
Cálculo del volumen	$V = \int_a^b A(x) dx$ $V = \int_a^b \pi (f(x))^2 dx$
Ecuación del resorte helicoidal	$\vec{r}(t) = \left( \cos t, \text{sen } t, \frac{t}{2\pi} \right)$
Derivada direccional	$D_{\hat{u}} f(x, y, z) = \nabla f(x, y, z) \cdot \hat{u}$ $\hat{u}$ : Vector unitario
Ecuación satisfecha por la carga de un circuito LRC	$Lq'' + Rq' + \frac{1}{C}q = E(t)$
Fuerza ejercida por un fluido	$F = \int_a^b \gamma y \cdot L(y) dy$
Fuerza que actúa sobre un líquido encerrado en un tubo	$F = \delta A 2x_0 g - \delta A 2xg$